Mutliphysics Modeling of Thin Layer Magnetoelectric Laminate Composites Using Shell Element

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***Abstract*— This paper proposes to use the shell element to model the thin magnetostrictive layers in finite element multiphysics modeling of magnetoelectric laminate composites. The multiphysics model includes the non-linearity of the thin layer magnetostrictive material and the electrical load effects of the ME device. The simulation results of a ME trilayer composite Metglas/PMN-PT/Metglas show the efficiency of the proposed method. This study provides the basis to study the ME devices composed of laminated thin layers.**

I. Introduction

The high magnetoelectric (ME) coefficients of artificial ME composites constituted by magnetostrictives and piezoelectric laminate layers make them ideal candidates for the design of modern potential applications such as magnetic field sensors, gyrators, variable inductances or energy transducers [1][2]. Recently, thanks to thin film deposit processes, the design of new ME composite materials [3] exhibiting giant ME coefficients have emerged by combining piezoelectric materials such as PMN-PT, PZT-4/5/8, or BTO with magnetostrictive materials such as CFO, FeONi, FeGa ou FeCoBaSi (Metglas).

Although in quasi-static approximation, the ME laminate composite materials can be modeled by homogenous methods such as magneto-elastic-electric equivalent circuits [4], they are not well adapted to include the magnetostriction nonlinearity or to consider the electro-mechanical impact when an electrical impedance load is connected between the electrodes of the piezoelectric layer. In these conditions, as shown in [5]-[7] the use of numerical code based on the finite element method code becomes essential. Nevertheless, the modeling of thin layers requires the use of extremely fine mesh that becomes very time consuming.

This paper proposes to alleviate this difficulty in introducing the shell element [8] into a 2D finite element formulation to consider thin layers in the simulation. The method is applied to simulate a ME trilayer composite Metglas/PMN-PT/Metglas.

II. Finite Element Formulation of the ME Composites

The 2D finite element formulation of an electro-magneto-elastic coupled problem including an electrical load connected to the electrodes of piezoelectric layer leads to solve in harmonic regime the matrix system (1) [6][7]:

$$[\mathbb{K}]\{\mathcal{X}\} = \{\mathcal{F}\} \tag{1}$$

with $[\mathbb{K}] = \begin{bmatrix} K_{uu} - \omega^2 M + j\omega C_{uu} & K_{up} & 0 & K_{ua} \\ K_{pu} & K_{pp} & K_{pq} & 0 \\ 0 & K_{qp} & -j\omega Z & 0 \\ K_{au} & 0 & 0 & K_{aa} \end{bmatrix}$,

$\mathcal{X} = [\boldsymbol{u} \quad V \quad Q \quad a_z]^T, \mathcal{F} = [f_{ex} \quad v_{ex} \quad q_{ex} \quad a_{ex}]^T$

where $[\mathcal{K}]$ is the electro-magneto-mechanical stiffness matrix, $[\mathcal{C}]$ the mechanical damping matrix, $[\mathcal{M}]$ the mechanical mass matrix, $\{\mathcal{X}\}$ the unknown vector and $\{\mathcal{F}\}$ the excitation vector in which the body force f_{ex}, the voltage v_{ex}, the electrical charge q_{ex} and the magnetic source a_{ex} are the external excitations. The unknown variables of the problem are the nodal displacement $\boldsymbol{u}$, electrical potential V, the electrical charge in electrical circuit Q and the magnetic vector potential a_z. In the case where the linear triangular elements are used, the submatrices in (1) can be formulated as [6][7][8] :

$$\begin{bmatrix} K_{uu} \\ K_{pp} \\ K_{aa} \end{bmatrix} = \sum_e \int_{\Omega_e} \begin{bmatrix} [G_u]^t \mathbb{c} [G_u] \\ [G_p]^t \mathbb{p} [G_p] \\ [G_a]^t \mathbb{v}\, [G_a] \end{bmatrix} d\Omega \tag{2-a}$$

and

$$\begin{bmatrix} K_{up} \\ K_{ua} \end{bmatrix} = \sum_e \int_{\Omega_e} \begin{bmatrix} [G_u]^t \mathbb{e}^t [G_p] \\ [G_u]^t \mathbb{g}^t [G_a] \end{bmatrix} d\Omega \tag{2-b}$$

$$K_{pu} = {K_{up}}^T, K_{au} = {K_{ua}}^T$$

$$K_{pq} = \begin{cases} \pm 1, \text{node} \in \ \text{input/output electrodes} \\ \qquad 0, \text{else where} \end{cases} \tag{2-c}$$

$$M = \rho_v \sum_e \int_{\Omega_e} [N]^t [N]\, d\Omega \tag{2-d}$$

$$C_{uu} = \beta K_{uu} + \alpha M \tag{2-e}$$

where $\mathbb{p}$ is the permittivity matrix, $\mathbb{e}$ the piezoelectric coefficients tensor, $\mathbb{v}$ the reluctivity matrix, $\mathbb{g} = \mathbb{q}\mathbb{v}$ the piezomagnetic coefficients tensor, $\mathbb{c}$ the elasticity tensor and ρ_v the mass density of the medium. It can be noted that $\mathbb{v}$ and $\mathbb{g}$ are incremental coefficients at a given magnetic bias determined by non-linear static analysis [6]. Ω_e denotes the element domain, and $G_u = \frac{1}{2}(grad_s + {grad_s}^T)[N]$, $G_V = grad_s[N]$, $G_a = rot_s[N] = r^* grad_s[N]$ with $r^* = [0 \quad 1, -1 \quad 0]^t$ the rotation matrix in Cartesian coordinates and N is the nodal shape function associate to each node. β and α represent the Rayleigh's mechanical damping coefficients.

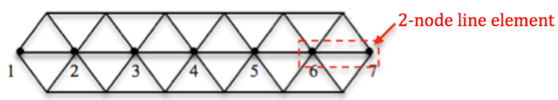
III. Modeling of Thin Magnetostrictive Layer

In the case of ME composite with thin/thick layers [3], the thickness difference between the magnetostrictive layer and the piezoelectric layer is very high. The modelling by the traditional finite elements requires the extremely fine mesh and is very time consuming. To alleviate the complexity, we introduce the shell elements to model the thin layers.

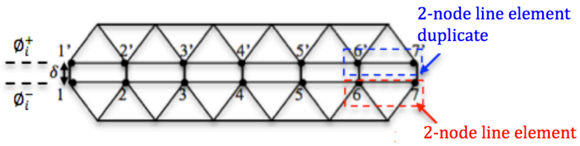
As represented in Fig. 1, we use the gradient approximation proposed in [8] to degenerate the 3-node linear triangular element to 2-node line element as follows:

$$grad_l[N\emptyset] = \frac{\partial}{\partial x}\left(\sum_{i=1}^{2} N_i^0 \langle\emptyset_i\rangle\right)\overrightarrow{e_x} + \frac{1}{\delta}\left(\sum_{i=1}^{2} N_i^0 [\emptyset_i]\right)\overrightarrow{e_y} \qquad (3)$$

where $\langle\emptyset_i\rangle = \frac{\emptyset_i^+ + \emptyset_i^-}{2}$ and $[\emptyset_i] = \frac{\emptyset_i^+ - \emptyset_i^-}{2}$ are respectively, the weighted average and the jump of unknown variable $\emptyset_i$ along and across the thickness δ. The annotations $\emptyset_i^+$ and $\emptyset_i^-$ represent the nodal values of $\emptyset_i$ on the two sides of the thin layer element and N_i^0 is the shape function for 2-node line element.



(a) Selection of a 2-node line element of a linear triangular mesh



(b) Shell nodal elements introduced in duplicate the 2-node line element

Fig.1. Method to include the thin layer

By considering the reluctivity, the piezomagnetic coefficients and the elasticity as constants along the thickness of the thin magnetostrictive shell layer, this last can be introduced in the simulation in employing the following new submatrices:

$$K_{aa} = \delta\nu_{11}\sum_e \int_{L_e} G_a^{shell^T} G_a^{shell}\, dl \qquad (4\text{-a})$$

$$K_{uu} = \delta c_{11}\sum_e \int_{L_e} G_u^{shell^T} G_u^{shell}\, dl \qquad (4\text{-b})$$

$$K_{ua} = \delta g_{11}\sum_e \int_{L_e} G_u^{shell^T} G_a^{shell} dl \qquad (4\text{-c})$$

$$M = \delta\rho_m\sum_e \int_{L_e} N_i^{0^T} N_i^0\, dl \qquad (4\text{-d})$$

where $G_u^{shell} = \frac{1}{2}\left(grad_l + grad_l^{\,T}\right)[N_i^0]$ and $G_a^{shell} = grad_l[N_i^0]$, ρ_v is the mass density of the magnetostrictive layer and L_e denotes the length of the 2-node line element.

IV. Example of Application

The example presented in Fig. 2 is a ME trilayer laminated composite Metglas/PMN-PT/Metglas in which the Metglas is magnetized along the longitudinal direction whereas the PMN-PT is polarized along the transversal direction (i.e. L-T mode). The thickness of Metglas is very thin compared to which of PMN-PT. Their dimensions are, respectively, L_x=10mm, t_p=2mm and δ =1µm. A resistive load Z=1 MΩ is connected to the up and down electrodes of PMN-PT. The excitation is an externally applied magnetic field $H_{ext.}$ To immobilize the device, two fixed mechanical displacements $u_x = u_y = 0$ are imposed on the middle (bottom and top) in each Metglas layers.

Fig. 3 and Fig. 4 show the field plots concerning respectively the total vector potential distribution a_z, the induced electric potential V and the nodal displacement $\boldsymbol{u}$ inside the composite at a given time.

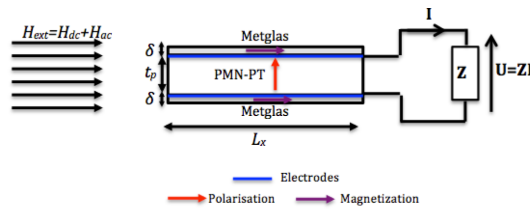
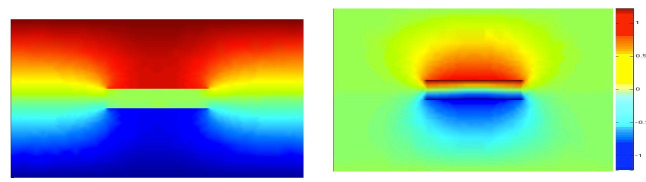


Fig. 2. A thin trilayer laminate composite in L-T mode



Total magnetic vector potential   Induced electric scalar potential

Fig. 3. Simulation distributions

Fig. 4. Displacement orientations inside the composite

V. Conclusion

This paper presents a method to model thin magnetostrictive layers using the shell nodal element in the finite element modeling of ME composites. This method is applied to simulate a trilayer composite with very thin magnetostricive layers. The detail of the procedure including the magnetostrictive nonlinarity as well as the frequency responses of the voltage coefficient under different thickness values and different impedance load values will be presented in the final version.

References

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